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## Corrections to low-frequency resistance measurements of self-heated metal microstructures

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**Abstract.** The resistance of self-heated mesoscopic structures has been used for some time as a way to probe the characteristics of electrons and phonons in these systems. In this paper we consider the differences between low-frequency dynamic and static resistance measurements of mesoscopic systems in the steady state. We present a general description of these non-equilibrium experiments, and discuss the connection to an integration procedure used to determine the differences between the two methods of resistance measurement. We have applied the analysis to electron-heating experiments and have shown that the difference between the static and dynamic resistance depends on the electron-phonon coupling and the acoustic coupling appropriate for the mesoscopic structure.

### 1. Introduction

Over the past decade non-equilibrium experiments have been performed on metal microstructures to study a variety of phenomena associated with the reduced dimensionality of electrons and phonons [1–3]. Frequently, these experiments use a DC electric current passing through the microstructure to transfer energy to the electron system, thereby increasing the electron temperature. The electron-phonon system reaches a steady state with a characteristic energy relaxation time that depends on the electron-phonon coupling in the microstructure and the acoustic coupling between the microstructure and the supporting substrate. This situation is well described by an energy bottleneck, with the weakest process dominating the overall energy relaxation [4, 5]. The bottleneck also determines the non-equilibrium acoustic phonon distribution in the microstructure.

To make a quantitative estimate of the energy relaxation time it is necessary to determine experimentally the steady-state value of the electron temperature. This has been achieved using thermometry based on, for example, Johnson noise [6], the Kondo effect [7], the Coulomb anomaly and weak localization [8]. In these phenomena, a temperature-dependent static resistance has been used to estimate the rise in the electron temperature as the DC electric current through the microstructure is increased. To enhance the signal-to-noise ratio, a low frequency AC bridge technique is often employed to measure the resistance. This is accomplished by applying a small low-frequency sensing current to the microstructure and using synchronous detection at the AC frequency. In this way the dynamic resistance,  $dV/dI$ , of the sample is measured as a function of the applied DC electric field. Obviously, if the sample is ohmic the static and dynamic resistances are the same. However, if the sample is non-ohmic, as expected in the case of self-heating due to the current through the sample, the dynamic and static resistances will differ. For metal samples of moderate disorder, the static resistance change due to the Coulomb anomaly [9] and weak localization [10] is usually small compared with the impurity resistance. Therefore, when estimating

the electron temperature using a low frequency AC measurement in the Coulomb anomaly and weak localization regimes, it is necessary to account for any small differences between the static and dynamic resistance that occur from self-heating by the AC and DC fields.

In this paper we discuss a general form of the difference between the dynamic and static measurement of resistance and show that for mesoscopic, disordered metal systems it depends on the electron-phonon scattering rate and the phonon escape time from the structures. We show that the heating can be understood as a trajectory on an electric field, temperature and current density surface. The present approach elucidates the physics underlying an integration procedure introduced by others to correct for the difference between the static and dynamic resistances [11, 12]. Indeed, under certain assumptions the integration procedure and the general approach are mathematically equivalent.

## 2. Model

We consider the use of an AC technique in the presence of a DC electric field to measure the resistance of a metal microstructure. For convenience we formulate the problem in terms of the conductivity rather than resistance. The DC current density,  $J$ , through the film is a function of the DC electric field,  $E$ , across the film and the electron and phonon temperatures,  $T_e$  and  $T_p$ , respectively, in the film. In this paper we assume the electron-electron scattering is sufficiently strong that  $T_e$  is well defined. The acoustic phonon distribution,  $n(\omega)$ , in the steady state experiments is not expected to be a Bose distribution [4, 5]. Equating  $n(\omega)$  to a Bose distribution a temperature can be assigned to each frequency,

$$T(\omega) = \hbar\omega/k_B \ln(n(\omega)^{-1} + 1). \quad (1)$$

These temperatures can be used to define the effective phonon temperature,

$$T_p = \int T(\omega)D(\omega)n(\omega) d\omega / \int D(\omega)n(\omega) d\omega \quad (2)$$

where  $D(\omega)$  is the acoustic phonon density of states.

In general,  $J = J(T_e, T_p, E)$  and the change in  $J$  can be written in terms of changes in the field and the temperature as

$$dJ = (\partial J/\partial T_e)_E dT_e + (\partial J/\partial T_p)_E dT_p + (\partial J/\partial E)_{T_e, T_p} dE. \quad (3)$$

The measured electrical conductivity can then be expressed as

$$dJ/dE = (\partial J/\partial T_e)_E dT_e/dE + (\partial J/\partial T_p)_E dT_p/dE + (\partial J/\partial E)_{T_e, T_p}. \quad (4)$$

If we make the formulation less general and write  $J = \sigma_s(T_e, T_p)E$ , then

$$\frac{dJ}{dE} = \sigma_s(T_e, T_p) + \left[ E \left( \frac{\partial \sigma_s(T_e, T_p)}{\partial T_e} \right)_{T_p} \frac{dT_e}{dE} + E \left( \frac{\partial \sigma_s(T_e, T_p)}{\partial T_p} \right)_{T_e} \frac{dT_p}{dE} \right]. \quad (5)$$

For brevity we call the term in brackets on the RHS of (5)  $\sigma_c$ .

When the DC field applied to the film is zero and the sensing current is kept very small so that the energy gained by the electrons is less than the thermal energy, the experiment is in equilibrium,  $T_e = T_p$  and  $\sigma_c \simeq 0$ , hence the measured resistance is the static resistance. When the DC field is increased from zero and the system has reached a steady state, the magnitude of  $\sigma_c$  will be greater than zero so that the AC and DC conductivities will differ.

The difference between the AC and DC conductivities depends on  $(\partial\sigma_s(T_e, T_p)/\partial T)$  and  $dT/dE$  for both the electron and phonon temperatures. To determine the quantities  $(\partial\sigma_s(T_e, T_p)/\partial T_e)_{T_p}$  and  $(\partial\sigma_s(T_e, T_p)/\partial T_p)_{T_e}$  we can use the measured equilibrium temperature dependence of the resistance. In general the static resistance can be written as

$$R(T_e, T_p) = \sum_i R_i(T_e, T_p) \quad (6)$$

where  $R_i(T_e, T_p)$  represent independent processes in the system. The coefficients of each  $R_i(T_e, T_p)$  can be obtained from the equilibrium temperature dependence of the measured resistance. The terms  $dT_e/dE$  and  $dT_p/dE$  are related to the amount of energy transferred to the electron system from the DC electric field. This problem has been discussed in detail elsewhere [13] and it has been shown that [4, 5]

$$J \cdot E = \sum_j \int \frac{\hbar\omega D_j(\omega)(n(\omega, T_e) - n(\omega, T_s))}{\tau_{\text{esc}}^j + \tau_{\text{ep}}^j(\omega)} d\omega \quad (7)$$

where  $D_j(\omega)$  is the phonon density of states for mode  $j$ ,  $n(\omega, T)$  is a Bose distribution evaluated at a temperature  $T$ ,  $\tau_{\text{esc}}^j$  is the time for phonons of mode  $j$  to escape from the film, and  $\tau_{\text{ep}}^j(\omega)$  is the electron-phonon scattering time for the corresponding mode. The relationship between the electron temperature,  $T_e$ , the substrate temperature,  $T_s$ , and  $E$  can be obtained after numerically solving (7). In the case of strong acoustic coupling between the microstructure and the substrate, and describing the electron-phonon coupling by the Pippard formalism [14], we have recently shown [13] that  $T_e \propto E^{2/5}$ , as predicted earlier [15]. Furthermore, we showed that the power law relationship between  $T_e$  and  $E$  depends on the strength of the acoustic coupling. For each value of  $T_e$  the steady-state phonon distribution  $n(\omega)$  can be calculated from which the function  $T_p(E)$  can be obtained. Hence, the non-ohmic behaviour of a mesoscopic system is related to the relaxation times  $\tau_{\text{esc}}^j$  and  $\tau_{\text{ep}}^j(\omega)$  through  $dT_e/dE$  and  $dT_p/dE$  and if the exact functional form of the relaxation times were known, (5) could be used to determine the relationship between the static and dynamic conductivities.

The connection between the static and dynamic conductivities in the non-ohmic system may also be determined using a procedure discussed by Payne *et al* [11] and more recently by Lane *et al* [12]. This procedure relies on the fact that for experiments in which the electrons are heated only by an applied DC electric field, the electron and phonon temperatures are only functions of  $E$ , from which  $J = \sigma_s(E)E$ . The measured dynamic conductivity is simply

$$dJ/dE = E d\sigma_s(E)/dE + \sigma_s(E) \quad (8)$$

where the first term on the RHS can be identified as  $\sigma_c$  which, as discussed above, is dependent on  $\tau_{\text{esc}}^j$  and  $\tau_{\text{ep}}^j(\omega)$ . Therefore, calculating the average value of the dynamic conductivity up to the applied field gives the static conductivity

$$\sigma_s(E) = (1/E) \int_0^E (dJ/dE) dE \quad (9)$$

from which it is possible to determine the trajectory describing the non-equilibrium electronic characteristics of the structure on the surface defined by  $J = \sigma_s(T_e, T_p)E$ .

### 3. Discussion

We have applied the analysis to the experiments of Nabity and Wybourne [16]. In these experiments a DC electric field was used to heat the electrons in  $1 \mu\text{m} \times 10 \mu\text{m} \times 250 \text{ \AA}$  Au-Pd films deposited on silicon substrates. The Coulomb anomaly has been shown to be the dominant low-temperature resistance correction in Au-Pd films [8] and was used to measure the electron temperature. Nabity and Wybourne studied the films in two situations: first, to provide good thermal contact to the film, the substrate and film were immersed in liquid helium; and second, the film was placed in a vacuum. For the present discussion we concentrate on the first experimental situation in which the acoustic coupling between the film and its surrounding is strong.

The static resistance of the films used in the experiments can be expressed as the sum of two terms

$$R(T_e, T_p) = R_{\text{int}}(T_e) + R_{\text{D}}(T_e, T_p) \quad (10)$$

where  $R_{\text{int}}(T_e)$  is a Coulomb anomaly term, and  $R_{\text{D}}(T_e, T_p)$  is a Drude term that includes electron-phonon and impurity scattering. In equilibrium the electrons and phonons in the microstructure are maintained at a common constant temperature  $T_s$ . The equilibrium resistance of the films as a function of  $T_s$  is shown in figure 1 and is expressed as  $\Delta R/R = (R(T) - R_{0\text{eq}})/R_{0\text{eq}}$  where,  $R_{0\text{eq}}$  is the minimum measured resistance. At temperatures below the minimum, the resistance rises logarithmically which is consistent with the Coulomb anomaly in a two-dimensional system [9]. Above the minimum, the resistance rise is described by a power law that originates from the increasing electron-phonon coupling strength as the temperature is increased. The Coulomb and Drude terms, and the total equilibrium resistance are shown by the dashed, dotted and solid curves, respectively, in figure 1.

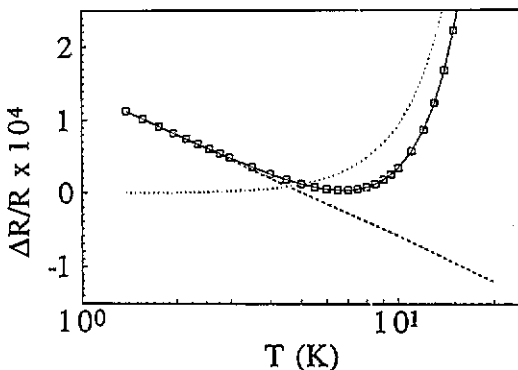


Figure 1. Fit to the equilibrium data of Nabity and Wybourne [16] using the Coulomb anomaly (---) and the Drude term (.....) contributions to the total resistance. The solid curve is the sum of the two contributions and the squares represent the experimental data.

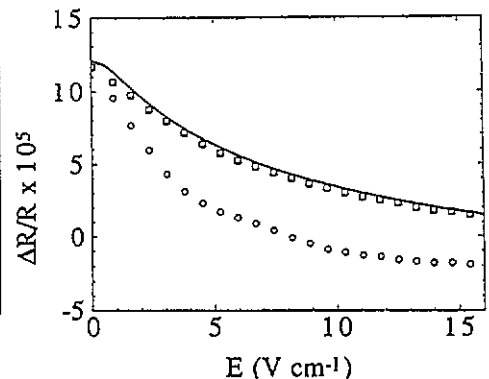


Figure 2. The circles show the measured dynamic resistance due to self-heating. The squares show the static resistance and the solid curve is the fit to the data.

When a DC electric field is applied to the film the electrons and the phonons warm up and reach different steady-state temperatures. In this situation the relative contribution to the total resistance from  $R_{\text{int}}(T_e)$  and  $R_{\text{D}}(T_e, T_p)$  is different than in equilibrium because each term has a different dependence on  $T_e$  and  $T_p$ . The measured dynamic resistance as

a function of  $E$  is shown in figure 2. To obtain the static resistance we have applied (9) to the data, the effect of which is seen in figure 2. The static resistance is then modelled using (1), (2) and (7), in which the adjustable parameters are  $\tau_{\text{esc}}^j$ ,  $\tau_{\text{ep}}^j(\omega)$ . Using a three-dimensional phonon density of states, the Pippard model to describe the electron-phonon interaction [14] and  $\tau_{\text{esc}}^j = d/v_j$  which is appropriate for perfect acoustic coupling, where  $d$  is the film thickness and  $v_j$  is the velocity of sound for mode  $j$ , we obtain a fit to the data shown by the solid line in figure 2. The material parameters used are the same as those reported previously [16], except the electron-phonon coupling strength has been increased by a factor of four. We note that such a straightforward fit to the data is not obtained for the film in vacuum. In this case it appears that the frequency dependence of the integrand in (7) needs to be weakened, which is consistent with the suggestion of reduced dimensionality for the phonons in thin films with weak acoustic coupling to the surroundings [1, 16].

From the fit to the data we can also obtain  $T_e(E)$  and  $T_p(E)$ , as shown in figure 3. From these relationships the function  $T_p(T_e)$  can be determined and the non-equilibrium problem can be reduced to  $J = \sigma_s(T_e)E$ . The steady-state characteristic of the film can then be obtained from (10) and describes a trajectory on the surface defined by  $J = \sigma_s(T_e)E$ , as shown in figure 4. We note that the equilibrium trajectory does not lie on the steady-state surface and so it is difficult to compare in a simple way the two trajectories.

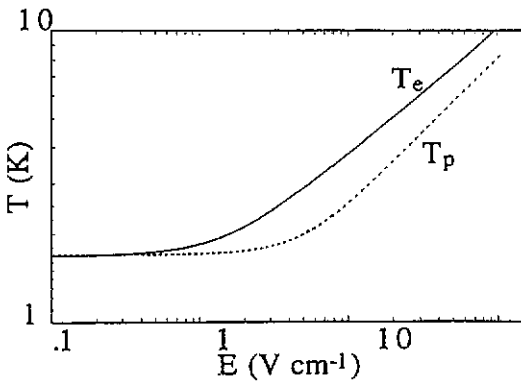


Figure 3. Relationship between electric field  $E$ , the electron temperature  $T_e$  and the phonon temperature  $T_p$ , calculated using the Pippard formalism [14] for the electron-phonon interaction and strong acoustic coupling between the film and the substrate.

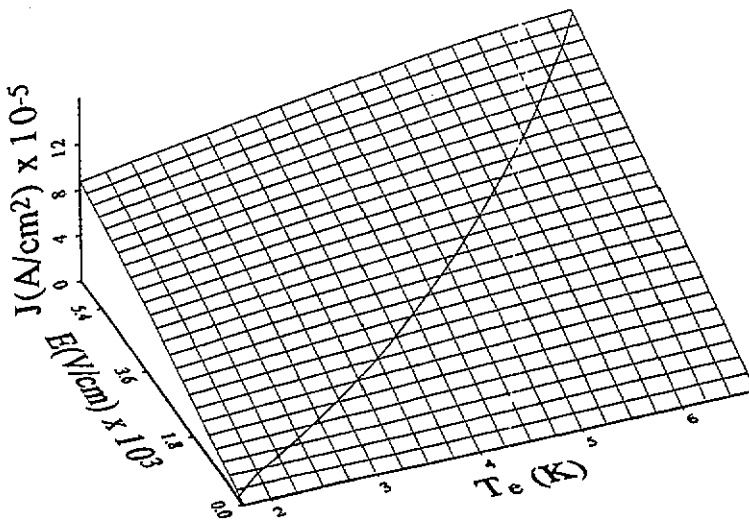


Figure 4. The steady-state surface defined by  $J = \sigma_s(T_e)E$ . The solid curve is the steady-state characteristic of the film. The field axis has been scaled to suppress the large constant term that comes from impurity scattering.

Finally, it is possible to compare the steady-state and equilibrium resistances by using the fact that the resistance  $R = R(T_e, T_p)$ . In this way we can plot a  $R, T_e, T_p$  surface and show the evolution of both the equilibrium and the static steady-state resistance as trajectories on the same surface, as shown in figure 5.

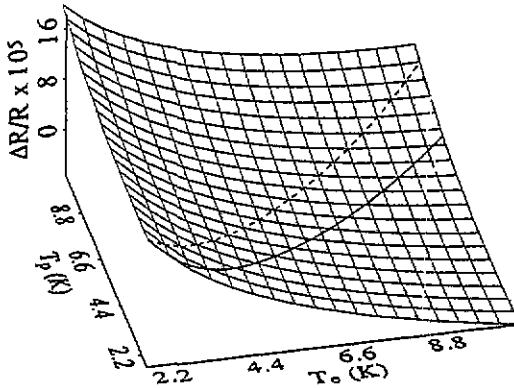


Figure 5. The evolution of the equilibrium and steady-state static resistances as a function of  $T_e$  and  $T_p$  are shown by the dashed and solid curves, respectively.

#### 4. Conclusion

We have shown that in the presence of a DC electric field the dynamic resistance measured by an AC technique can be used to determine the static resistance of a microstructure. The size of the non-ohmic effects in the microstructure are related to the electron-phonon coupling and the acoustic coupling of a particular system. The analysis we have presented is general and, therefore, will be applicable to a calculation of the effects of self-heating in other systems.

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